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A Novel Approach for Solving the Power Flow Equations

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ABSTRACT
This paper presents a detailed investigation into the effectiveness of iterative methods in solving the linear system problem in power flow solution process. Previously Newton method employing an LU method, GMRES method has been one of the most widely used power flow solution algorithms A fast Newton-FGMRES method for power flow calculations is proposed in this paper. Three accelerating schemes to speed up the Newton-FGMRES method are proposed.. The simulation result gives the effectiveness of proposing one compared with existing methods.

Index Terms: **Flexible GMRES method, iterative methods, Newton power flow calculation.**

I. INTRODUCTION
A majority of computational effort in the Newton power flow method lies in solving a set of linear equations. The traditional direct LU factorization method has been popular in solving a set of linear equations. In the last 20 years or so, the iterative methods emerged as a vital alternative to the traditional Newton-LU method due to its speed. However, direct methods find the exact solution after a finite number of steps. Iterative methods, on the other hand, successively approximate the solution to a predetermined degree of accuracy based on an initial guess.

For a set of very large linear equations, the use of direct methods is impractical; it simply takes too long. Experience in solving VLSI circuit design problems has confirmed the impracticality of LU factorization for large circuit design problems. The cost of using direct methods to solve a system of linear equations is of the order for dense matrices and to for sparse matrices. Stationary iterative methods bring the cost down to the order of for dense matrices and for sparse matrices.

Non stationary iterative methods, such as Krylov subspace methods [1], [2], converge in at most iterations (assuming no round-off error), where the system size, and preconditioning often significantly is reduces the required number of iterations. The benefits of using iterative methods over direct methods increase with system size. While both iterative methods and direct methods are applicable to solve small systems of linear equations, it is often hard to solve very large linear equations without using an iterative method. Direct methods take longer computation time for large-scale systems and this difficulty can be greatly improved by the use of iterative methods. It is important to note that the distinction between direct and iterative methods is becoming more blurred, as many preconditioning techniques result in methods that are a combination of both iterative and direct solvers. Nevertheless, there is still much to be learned from both methods.

The advantages of iterative solvers over direct methods based on the direct LU factorization method in power system applications have been demonstrated in [3]–[11]. It is now recognized by many researchers that the Newton-GMRES (generalized minimal residual) method can outperform the Newton-LU method when solving large-scale power flow equations. A significant amount of speed-up, for instance 50%, obtained by the Newton-GMRES method over the Newton-LU method has been achieved. Nonstationary/Krylov subspace methods have become more complicated because the operations performed at each step involve iteration dependent coefficients. The oldest Krylov subspace method is the Conjugate Gradient (CG) method for symmetric positive definite (SPD) matrices. Since the discovery of the oldest Krylov subspace method, the Conjugate Gradient method, much work has been done to find similar methods that can be applied to nonsymmetric and/or nondefinite matrices. Some of these newer, more general methods include the GMRES method, the Biconjugate Gradient Stabilized method, and the Quasi-Minimal Residual method. These methods are all termed Krylov subspace methods because they are derived with respect to a Krylov basis.

For iterative Krylov subspace methods, it has been found that preconditioning plays an important

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role in the convergence rate of iterative solvers. Several preconditioners developed for power system computations appeared in [5], [7], and [12]. However, these preconditioners for "normalizing" linearized power mismatch equations were fixed at each Newton iteration. Recently, an adaptive preconditioner was proposed for the Jacobian-free Newton-GMRES(m) method in [13]. The proposed preconditioners were updated using a rank-one update algorithm. However the updated preconditioners were only used for the linearized equations of next iterations. The preconditioners were still kept constant while solving the linear equations.

To further improve the iterative methods, a flexible inner outer Krylov subspace method (FGMRES, flexible inner-outer preconditioned GMRES) was developed [14], [15]. Different from the traditional iterative Krylov subspace methods, the preconditioners used in this FGMRES method were allowed to vary with in each iteration. Thus, the FGMRES method has been observed to be more effective than the traditional GMRES in several numerical studies [14].

In this paper, the FGMRES method is applied to solve linear equations arising from the Newton power flow method. To further improve the speed of this Newton-FGMRES method, three accelerating schemes are developed and incorporated into the proposed Newton-FGMRES. This paper compares the convergence characteristics and computational speed of the Newton-FGMRES and fast Newton-FGMRES with the traditional Newton-GMRES on two practical power systems: a 12 000-bus system and a 21 000-bus system. Numerical studies show the advantages of the proposed fast Newton-FGMRES in computational speed and in robustness under different loading conditions. We point out that the traditional direct method (Newton-LU) was used as a benchmark method for both the traditional Newton-GMRES method and the fast Newton-FGMRES method. We have also evaluated the fast decoupled Newton method on the two large-scale power systems. However, the fast decoupled Newton method diverges on both test systems.

2.1. PROBLEM FORMULATION

It is assumed that all control devices remain fixed throughout the Newton solution process. Hence, voltage regulating generators will be considered as PV buses with unlimited reactive capabilities. The power flow Jacobian will be formulated in polar coordinates, as follows A single iteration of the Newton process involves solving equation (1) for the state update **(A@,+)** and then updating the state vector *(0, V)*. Traditionally, the linear system (1) is solved via an LW factorization of the Jacobian, a forward elimination and a backward substitution. When solving the power flow equations, the Jacobian is relatively inexpensive to evaluate, since evaluation of the bus power mismatches involves similar calculations. Likewise, the forward elimination and backward substitution procedures are fairly inexpensive due to efficient sparse storage of the matrix factors *L* and *U*. Based on the UNIX run-time profiler output, the most time-consuming procedure of a single Newton iteration is the LU factorization of the Jacobian matrix. For large-scale power systems (e.g., the 3493 bus case studied here), the cost of an LU factorization of the system Jacobian dominates the costs of the other operations, consuming approximately **85%** of the total Newton process execution time.

2.2. NEWTON METHODS:

Nonlinear algebraic systems of equations are usually solved by a Newton method due to the local quadratic convergence. While this local contraction property is desirable, it is often the case that the last step of the Newton method decreases the residual of the nonlinear system well beyond the user specified tolerance. This "over solving" cannot be avoided in an exact Newton method when the linear system is solved directly via an LU factorization. However, an inexact Newton method, such **as** Newton- GMRES, monitors the level of accuracy in the solution by keeping track of the norm of the residual. Hence, an inexact Newton method based on an iterative linear solver can be stopped during the solution of the linear system, if the solution to the linear system has been computed accurately enough. By avoiding the waste of computation spent on over solving, an inexact Newton approach can be a serious competitor to a exact Newton method.

3. INEXACT NEWTON METHODS

An alternative to the direct solution (via LU factorization) of the linear system (1) is an iterative approach. Non stationary iterative methods for the solution of linear equations have received great attention recently from researchers in the field of numerical analysis. A promising technique in the category of Krylov subspace approaches is the Generalized Minimal Residual (GMRES [12]) method, which attempts to solve the linear system

$$Ax = b \qquad (2)$$

by minimizing the residual ***r*** defined by

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$$r(x) \stackrel{\text{def}}{=} b - Ax \qquad (3)$$

via Krylov subspace updates to the candidate linear system solution z. GMRES is a member of the family of Krylov subspace iterative methods, which

produces a sequence ***xk*** of approximations to the solution ***2*** = ***A-lb*** of linear system (2). In general, the Krylov subspace iterates are described by

$$x_k \in x_0 + \mathcal{K}_k(r_0, A), \qquad k = 1, 2, \ldots \qquad (4)$$

where z_o is the initial estimate of the solution to (2)

$$\mathcal{K}_k(r_0, A) = \operatorname{span}(r_0, Ar_0, \ldots, A^k r_0). \qquad (5)$$

In particular, GMRES creates a sequence ***2k*** that minimizes the norm of the residual at step ***k*** over the lcth Krylov subspace as follows

$$\|b - Ax_k\|_2 = \min_{x \in x_0 + \mathcal{K}_k(r_0, A)} \|b - Ax\|_2. \qquad (6)$$

At step *k* , GMRES applies the Arnoldi process to a set of *k* ortho normal basis vectors for the kth Krylov subspace to generate the next basis vector. When the norm Fig **1:** The GMRES(m) algorithm (for ***A*** E **PX")** without preconditioning of the newly created basis vector is sufficiently small, GMRES solves the following ***(k*** + **1)** x ***IC*** least squares problem

$$\|g_k - H_k y_k\|_2 = \min_{y \in C^k} \|g_k - H_k y\|_2,$$

where ***Hk*** is a ***(k*** + **1)** x ***k*** upper Hessenberg matrix of full rank ***k*** and ***gk*** = ***Ilrollel*** with standard basis vector **el** Ei ***Rk+l.*** 'Io solve the least squares problem, a Modified Gram- Schmidt procedure is generally used. We have described a restarted GMRES algorithm, following **[7],** in Figure 1. As mentioned in [7], a forward difference approximation can be used to compute the directional derivatives used by GMRES. Since the Jacobian matrix is only used by GMRES in matrix vector multiplications, it is possible to avoid the cost of creating the Jacobian matrix. However, the forward difference approximations to the directional derivatives involve evaluating the nonlinear power flow mismatch function at every GMRES iteration. However problems in large scale power systems present research area in terms of applicability still cannot compete with direct methods because of possible convergence problem.

Figure 1: Standard GMRES algorithm with right preconditioning

1. $r_0 = b - Ax_0,\ k = 0,\ \rho = \|r\|_2,\ v_1 = r_0/\rho$
   $errtol = \max(abstol, reltol\|b\|_2)$
   $g = \rho(1, 0, 0, \ldots) \in R^{m+1}$
2. While $\rho > errtol$ and $k < m$ do
   (a) $k = k + 1$
   (b) $v_{k+1} = Av_k$
   (c) for $j = 1, \ldots, k$
      i. $h_{j,k} = v_{k+1}^T v_j$
      ii. $v_{k+1} = v_{k+1} - h_{j,k} v_j$
   (d) $h_{k+1,k} = \|v_{k+1}\|_2$
   (e) $v_{k+1} = v_{k+1}/h_{k+1,k}$
   (f) Apply and create Givens rotations
      i. If $k > 1$ apply Q_{k-1} to the kth column of H
      ii. $\nu = \sqrt{h_{k,k}^2 + h_{k+1,k}^2}$
      iii. $c_k = h_{k,k}/\nu; \quad s_k = h_{k+1,k}/\nu$
      $h_{k,k} = c_k h_{k,k} - s_k h_{k+1,k}; \quad h_{k+1,k} = 0$
      iv. $g = G_k(c_k, s_k) g$
   (g) $\rho = |(g)_{k+1}|$
3. Set $r_{i,j} = h_{i,j}$ for $1 \le i, j \le k$
   Set $(w)_i = (g)_i$ for $1 \le i \le k$
   Solve the upper triangular system $Ry_k = w$
4. $x = x_0 + V_k y_k$

4. FAST NEWTON-FGMRES

The proposed fast Newton-FGMRES method is composed of the 1) Newton method, 2) FGMRES method for solving the linear equations, and 3) the three accelerating schemes including a hybrid scheme, a partial preconditioner update scheme, and an adaptive tolerance control scheme. The hybrid scheme generates the preconditioners for the inner iterations of the FGMRES method based on the complete LU factorization of the coefficient matrices. Of course, the complete LU factors can also be used to solve the corresponding linear equations.

When the dimension of the coefficient matrix changes, the preconditioner can be fast updated from the previous one by using the partial preconditioner update scheme. Using the adaptive tolerance control scheme, the stopping criterion used by FGMRES is based on the residuals from the previous Newton iterations. We are now in a position to present the fast Newton-FGMRES power flow method.

Step 1) Input the data of the power system to be studied.

Step 2) Initialization

i. Set the initial value for bus voltage.

ii. Construct the admittance matrix.

iii. Save the initial bus state information (PV bus or PQ bus) as flag
iv. Define a threshold value for preconditioner updates.
Step 3) Construct the Jacobian matrix J and evaluate the power mismatch ds
Step 4) Solve the power mismatch equation $J\Delta x = dS$. If L and U have not been formed, then
i. Set flag0=flag
ii. Factorize J and save the factors L,U: .
iii. Solve $J\Delta x = dS$ by a forward elimination and a backward substitution using L and U.
iv. Go to Step 5.
Else
i. If, Flag0≠Flag update the preconditioner by the partial preconditioner update scheme.
ii. Solve $J\Delta x = dS$ by the FGMRES method.
iii. If FGMRES converges to the tolerance of , go to Step 5. Otherwise, clear L and U do this step again.
Step 5) Update the bus voltage value.
$x = x + \Delta x$
Step 6) Check the reactive generation constraints and save the current bus state information as Flag.
Step 7) Decide whether a new preconditioner is needed.
i. Compare Flag with Flag0 and evaluate the number of buses whose states have been considerably changed m, .
ii. If,m≥m0then a new preconditioner is required. Clear L and U. Otherwise, keep L
and U.
Step 8) Stopping criterion: If , $\|\Delta x\|_2 \geq \varepsilon$ then go to step 3; otherwise, power flow calculation stops.

5. NUMERICAL RESULTS

The proposed Newton-FGMRES and fast Newton-FGMRES methods are evaluated on the following two practical power systems in North America: a 12 000-bus system and a 21 000-bus system. The traditional Newton-LU method, the Newton-GMRES method, the proposed Newton-FGMRES method, and the proposed fast Newton-FGMRES method are compared in terms of convergence characteristics and computation time. The initial guess for the iterative solver was selected to be a flat start. The convergence criterion was set to 10^{-5} and the maximum iteration number for GMRES and FGMRES was set to be 10. ILU- preconditioners were used in the Newton-GMRES method and the Newton-FGMRES method. Different parameters for ILU- were also considered: k=15in (a) and k=25 in (b). We also use the approximate minimum degree ordering as the sparse ordering scheme. We first considered the cases with unlimited Q-generation and then

TABLE I
COMPUTATION TIME FOR NEWTON POWER FLOW CALCULATION IN SECONDS FOR A 12 000 BUS SYSTEM BY DIFFERENT METHODS

	Newton-GMRES	Newton-FGMRES		Fast Newton-FGMRES	
		Time	Improvement	Time	Improvement
(a)	0.186	0.170	+8.6%	0.111	+40.3%
(b)	0.152	0.141	+7.2%		+27.0%

TABLE II
COMPUTATION TIME FOR NEWTON POWER FLOW CALCULATION IN SECONDS FOR A 21 000 BUS SYSTEM BY DIFFERENT METHODS

	Newton-GMRES	Newton-FGMRES		Fast Newton-FGMRES	
		Time	Improvement	Time	Improvement
(a)	0.518	0.489	+5.6%	0.422	+18.5%
(b)	0.472	0.461	+2.3%		+10.6%

considered the cases with limited Q-generation. In these numerical studies, the control actions of ULTC and phase-shifters are neglected. The computer used for the tests is described as follows: CPU: 1.83 GHz, the number of cores in CPU: 1, main memory: 1G, programming language: Fortran. All computation times shown in the tables are the average time. Note that we are only concerned with the computation time of the iterative process. For the fast Newton-FGMRES method, the computation times listed in the tables correspond to those required from Step 3 to Step 8 of the flowchart of the method.

We have observed that both the Newton-FGMRES method and the fast Newton-FGMRES method converge faster than the Newton-GMRES method on the 12 000-bus system and the 21 000-bus system, respectively. The total computation time required by the three methods is summarized in Tables I and II. The proposed fast Newton-FGMRES method is generally faster than the Newton-FGMRES method and the Newton GMRES method. The difference in required computation time can be significant. For the 12 000-bus system, the fast Newton-FGMRES method can be 40.3% faster than the traditional Newton-GMRES method. For the 21 000-bus system, the fast Newton-FGMRES method can be 18.5% faster than the traditional Newton-GMRES method. This reveals that the three accelerating schemes are effective in improving the performance of the Newton-FGMRES. Figs. 1 and 2 shows the convergence characteristics of the three methods on the 12 000-bus system and the 21 000-bus system, respectively. It can be observed that by the use of FGMRES, the Newton iteration can be reduced by at least two times the iterations using GMRES. Therefore both the Newton-FGMRES method and the fast Newton-FGMRES method converge faster than the traditional Newton-GMRES method. We next considered the cases with limited Q-generation. The upper and lower limits of the reactive power generation were checked during the solution process. The proposed fast Newton-FGMRES method was compared with the traditional Newton- GMRES method in terms of computation time. The test results are summarized in Table III The traditional direct method (Newton-LU) was used as a benchmark method for both the traditional Newton-GMRES method and the fast Newton FGMRES method. The fast refactorization method used in this Newton-LU method is described as follows. The factorization is divided into two steps: the symbol decomposition and the numerical decomposition. In the symbol decomposition, the positions of nonzero fill-ins are identified

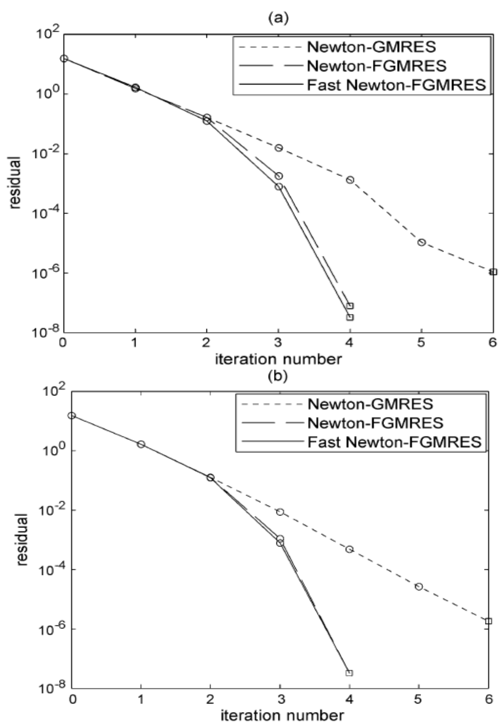

Fig. 2. Convergence characteristics of different methods on the 12 000-bus system.

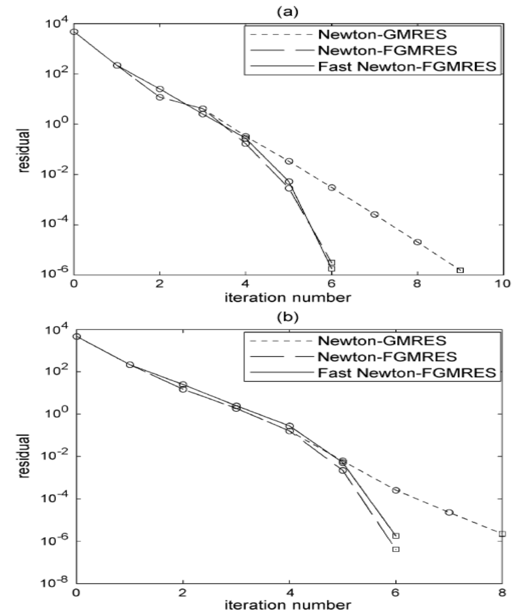

Fig 3. Convergence characteristics of different methods on the 21 000-bus

Our numerical studies reveal that the Newton-GMRES method has no superiority in speed over the traditional Newton-LU method on these two large-scale power systems. This can be explained from the viewpoint of the precondition: the dimension of the Jacobian changes during the iterative process and the repeating construction of preconditioners damages the advantage of the traditional Newton-GMRES method. However, the proposed fast Newton-FGMRES method can still be faster than the Newton-LU power flow method by 16.6% on the 12 000-bus system and 26.2% on the 21 000-bus system. In light of numerical evaluations of these two large-scale power systems, it may be concluded that the proposed fast Newton-GMRES method is considerably faster than the traditional Newton-LU method system.

TABLE III
COMPUTATION TIME OF NEWTON POWER FLOW IN SECONDS BY DIFFERENT SOLVERS

Bus Number	Newton -LU	Newton-GMRES		Fast Newton-FGMRES	
		Time	Improvement	Time	Improvement
12000	0.307	0.322	-4.9%	0.256	+16.6%
21000	0.836	0.844	-1.0%	0.617	+26.2%

Different Loading Conditions:

TABLE IV
COMPUTATION TIME OF THE NEWTON POWER FLOW IN SECONDS AT DIFFERENT LOADING CONDITIONS FOR A 21 000 BUS SYSTEM BY THE NEWTON LU AND THE FAST NEWTON-FGMRES

Loading Condition	Newton-LU	Fast Newton-FGMRES	
		Time(s)	Improvement
0.5	0.642	0.436	+32.0%
0.6	0.642	0.418	+34.9%
0.7	0.642	0.418	+34.8%
0.8	0.642	0.418	+34.9%
0.9	0.643	0.418	+34.9%
1.0	0.777	0.422	+45.7%
1.1	0.771	0.448	+41.9%
1.2	diverge	diverge	----

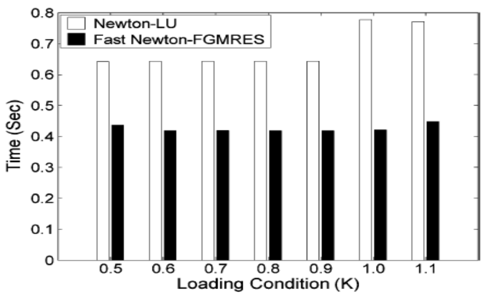

Fig4.Comparison of the computation time required by the Newton-LU and the fast Newton-FGMRES under different loading conditions on the 21 000 bus system.

TABLE V
MAXIMUM AND MINIMUM COMPUTATION TIME REQUIRED BY THE NEWTON-LU AND FAST NEWTON-FGMRES ON THE 21 000-BUS SYSTEM

	Newton-LU		Fast Newton-FGMRES	
	K	Time(Sec)	K	Time(Sec)
Maximum Time	1	0.777	1.1	0.448
Minimum Time	0.5	0.642	0.8	0.418
Difference	21.0%		7.2%	

VI. CONCLUSIONS

In this paper, we have proposed a Newton-FGMRES method for solving power flow equations. From a computational viewpoint, Newton-FGMRES is a slight extension of the existing Newton-GMRES method. However, we have explored the numerical characteristics of power flow equations and developed three accelerating schemes including a hybrid scheme, The proposed fast Newton-FGMRES solver has been evaluated on two practical large-scale power systems, one with 12 000 buses and another with 21 000 buses. Numerical results show the advantages of the proposed fast Newton-FGMRES method as opposed to the traditional Newton-GMRES method in terms of the convergence characteristics and the computation time. Even though the Newton-GMRES method has no superiority in speed over the traditional Newton-LU method on these two large-scale power flow equations, the proposed fast method consistently outperforms both the traditional Newton-LU and Newton-GMRES in terms of computational speed.

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